FLUID MECHANICS
(BTME 301-18)

Unit 5: Dimensional Analysis And Similitude

Introduction. DIMENSIONS AND UNITS

- A dimension is a measure of a physical quantity (without numerical values), while a unit is a way to assign a number to that dimension. For example, length is a dimension that is measured in units such as microns (μm), feet (ft), centimeters (cm), meters (m), kilometers (km), etc.
- There are seven primary dimensions (also called fundamental or basic dimensions)—mass, length, time, temperature, electric current, amount of light, and amount of matter.
- All nonprimary dimensions can be formed by some combination of the seven primary dimensions.
- For example, force has the same dimensions as mass times acceleration (by Newton's second law). Thus, in terms of primary dimensions,

$$\text{Dimensions of force:} \quad \{\text{Force}\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\right\} = \{mL/t^2\}$$

Primary dimensions and their associated primary SI and English units

Dimension	Symbol*	SI Unit	English Unit
Mass	m	kg (kilogram)	lbm (pound-mass)
Length	L	m (meter)	ft (foot)
Time†	t	s (second)	s (second)
Temperature	T	K (kelvin)	R (rankine)
Electric current	I	A (ampere)	A (ampere)
Amount of light	C	cd (candela)	cd (candela)
Amount of matter	N	mol (mole)	mol (mole)

- Surface tension (σ_s), has dimensions of force per unit length. The dimensions of surface tension in terms of primary dimensions is

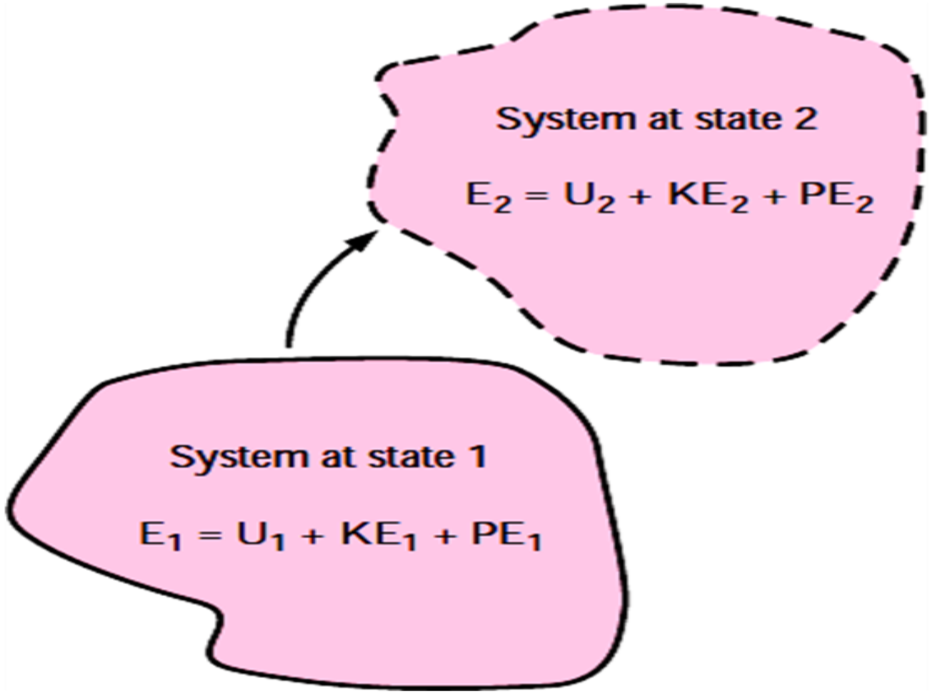
$$\text{Dimensions of surface tension:}\quad \{\sigma_s\} = \left\{\frac{\text{Force}}{\text{Length}}\right\} = \left\{\frac{\text{m}\cdot\text{L/t}^2}{\text{L}}\right\} = \{\text{m/t}^2\}$$

DIMENSIONAL HOMOGENEITY

- **Law of dimensional homogeneity:** Every additive term in an equation must have the same dimensions.
- Consider, for example, the change in total energy of a simple compressible closed system from one state and/or time (1) to another (2), as shown in the figure
- The change in total energy of the system (ΔE) is given by

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

- where E has three components: internal energy (U), kinetic energy (KE), and potential energy (PE).



DIMENSIONAL HOMOGENEITY

- These components can be written in terms of the system mass (m); measurable quantities and thermodynamic properties at each of the two states, such as speed (V), elevation (z), and specific internal energy (u); and the known gravitational acceleration constant (g),

$$\Delta U = m(u_2 - u_1) \qquad \Delta KE = \frac{1}{2}m(V_2^2 - V_1^2) \qquad \Delta PE = mg(z_2 - z_1)$$

- It is straightforward to verify that the left side of the change in Energy equation and all three additive terms on the right side have the same dimensions—energy.

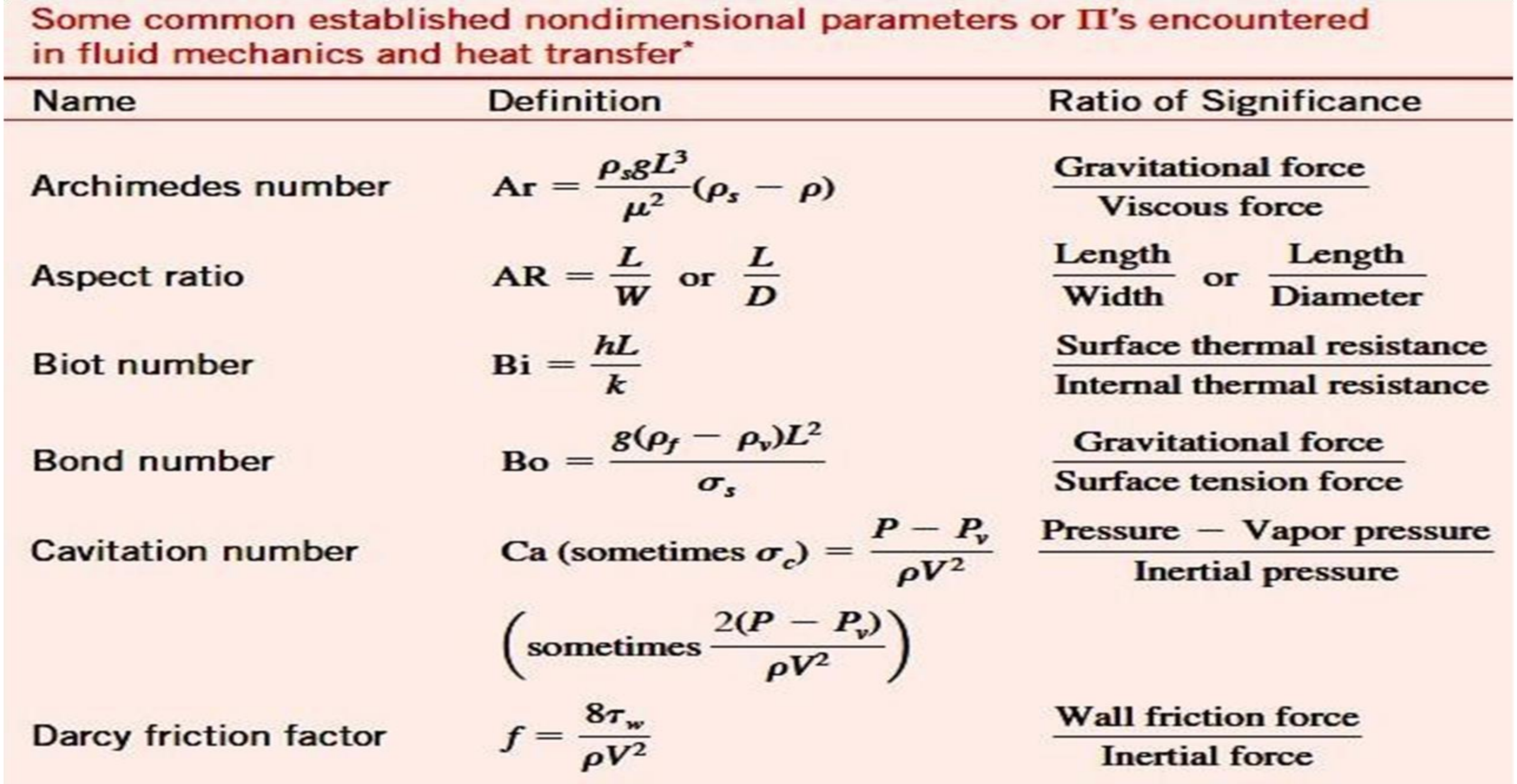
$$\{\Delta E\} = \{\text{Energy}\} = \{\text{Force} \cdot \text{Length}\} \quad \rightarrow \quad \{\Delta E\} = \{mL^2/t^2\}$$

$$\{\Delta U\} = \left\{\text{Mass}\,\frac{\text{Energy}}{\text{Mass}}\right\} = \{\text{Energy}\} \quad \rightarrow \quad \{\Delta U\} = \{mL^2/t^2\}$$

$$\{\Delta KE\} = \left\{\text{Mass}\,\frac{\text{Length}^2}{\text{Time}^2}\right\} \quad \rightarrow \quad \{\Delta KE\} = \{mL^2/t^2\}$$

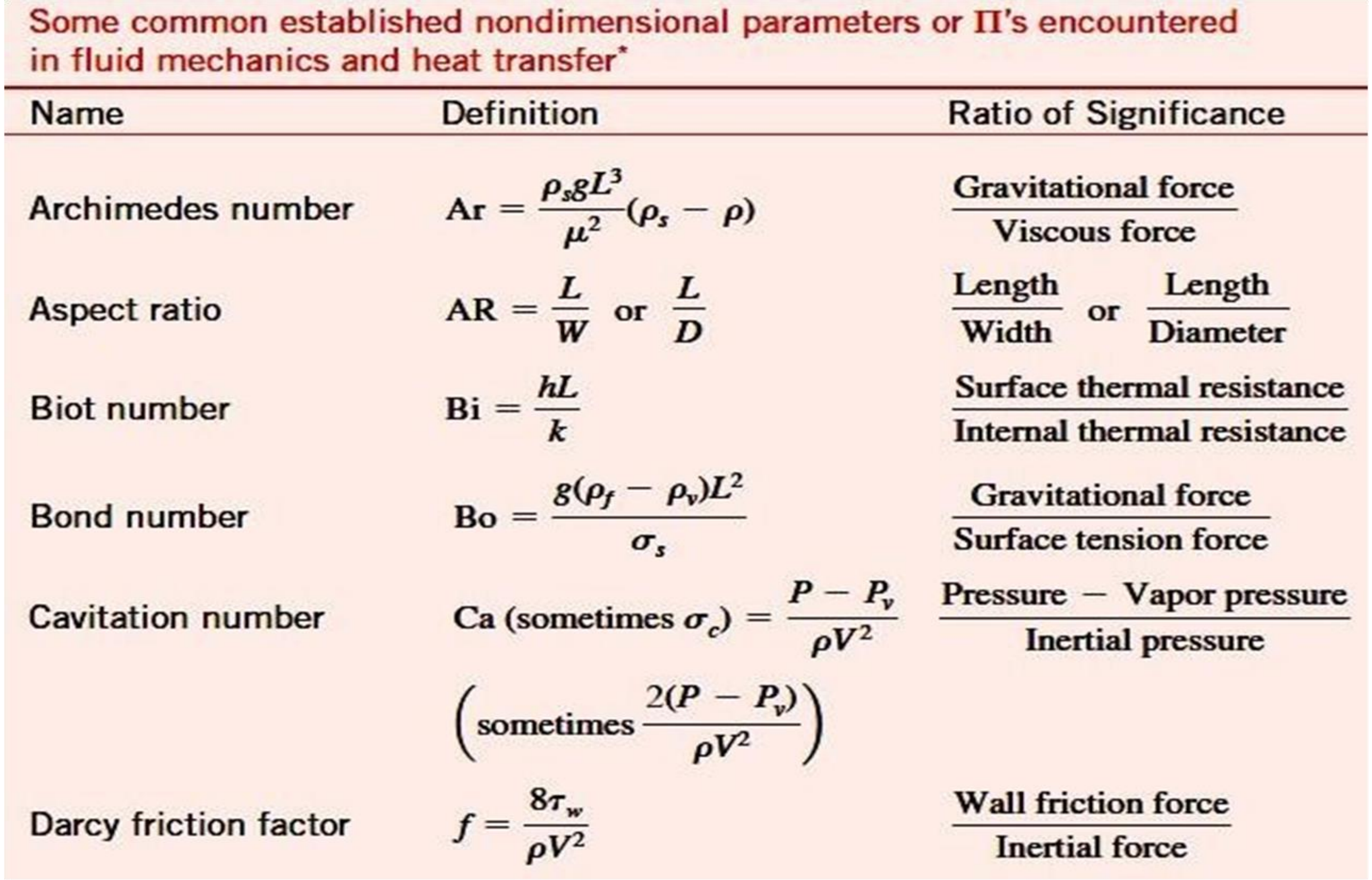
$$\{\Delta PE\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\,\text{Length}\right\} \quad \rightarrow \quad \{\Delta PE\} = \{mL^2/t^2\}$$

- In addition to dimensional homogeneity, calculations are valid only when the units are also homogeneous in each additive term.
- For example, units of energy in the above terms may be J, N·m , or kg·m^2/s^2, all of which are equivalent.
- Suppose, however, that kJ were used in place of J for one of the terms. This term would be off by a factor of 1000 compared to the other terms.
- It is wise to write out all units when performing mathematical calculations in order to avoid such errors.



Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Archimedes number	$\text{Ar} = \dfrac{\rho_s g L^3}{\mu^2}(\rho_s - \rho)$	$\dfrac{\text{Gravitational force}}{\text{Viscous force}}$
Aspect ratio	$\text{AR} = \dfrac{L}{W}$ or $\dfrac{L}{D}$	$\dfrac{\text{Length}}{\text{Width}}$ or $\dfrac{\text{Length}}{\text{Diameter}}$
Biot number	$\text{Bi} = \dfrac{hL}{k}$	$\dfrac{\text{Surface thermal resistance}}{\text{Internal thermal resistance}}$
Bond number	$\text{Bo} = \dfrac{g(\rho_f - \rho_v)L^2}{\sigma_s}$	$\dfrac{\text{Gravitational force}}{\text{Surface tension force}}$
Cavitation number	$\text{Ca (sometimes } \sigma_c) = \dfrac{P - P_v}{\rho V^2}$ $\left(\text{sometimes } \dfrac{2(P - P_v)}{\rho V^2}\right)$	$\dfrac{\text{Pressure} - \text{Vapor pressure}}{\text{Inertial pressure}}$
Darcy friction factor	$f = \dfrac{8\tau_w}{\rho V^2}$	$\dfrac{\text{Wall friction force}}{\text{Inertial force}}$



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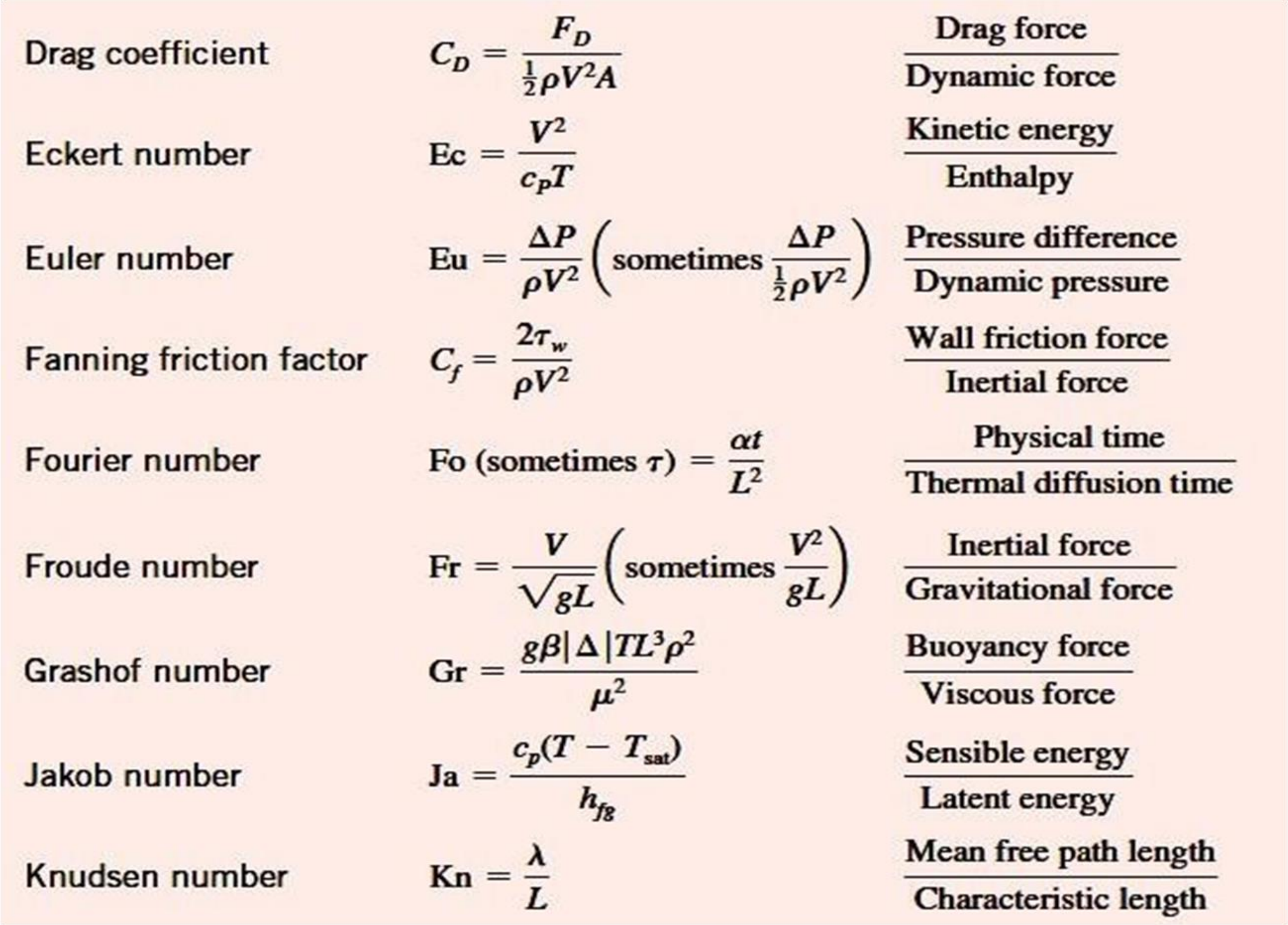
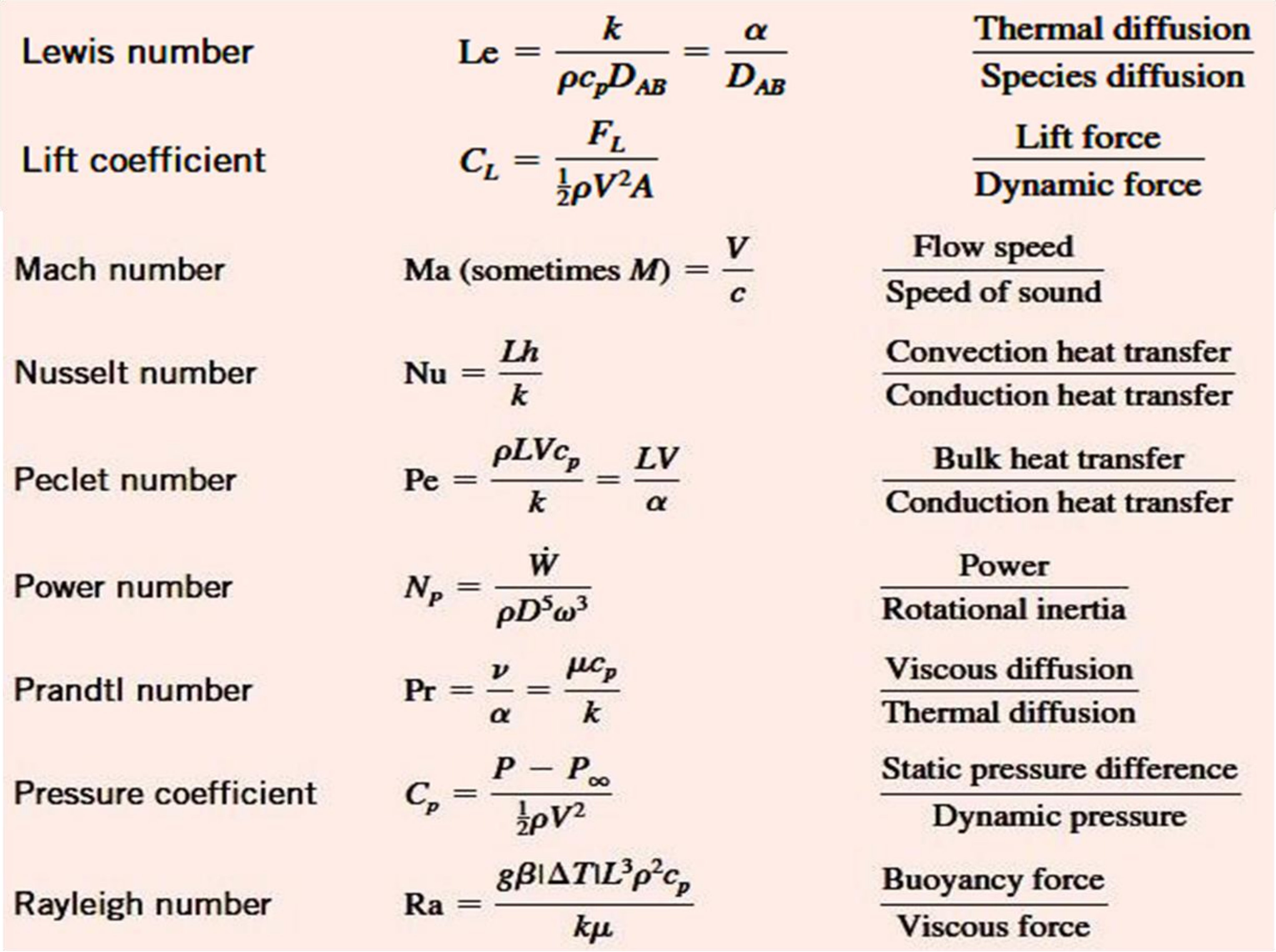
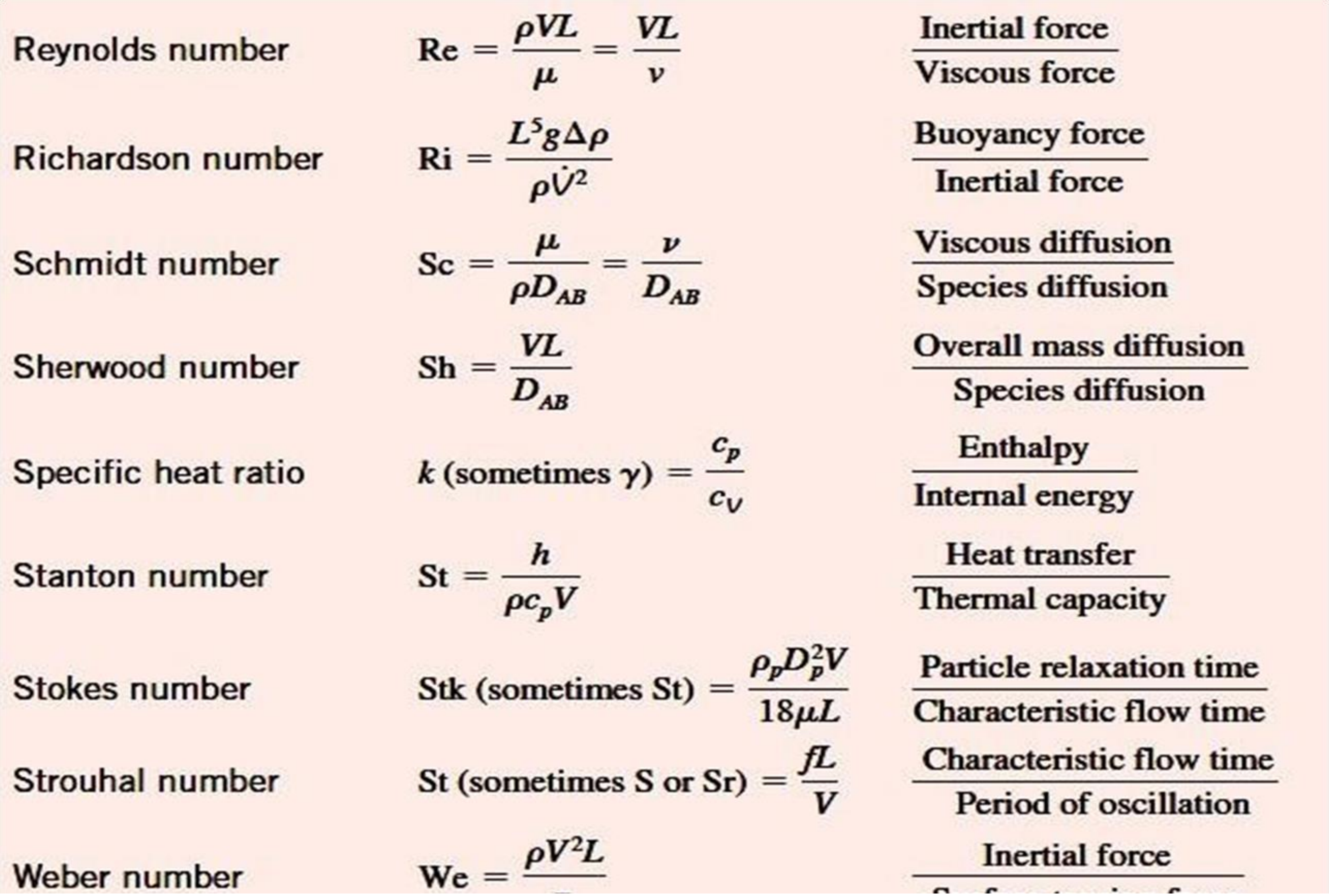


Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$
Eckert number	$\text{Ec} = \frac{V^2}{c_p T}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$\text{Eu} = \frac{\Delta P}{\rho V^2} \left(\text{sometimes } \frac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\frac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \frac{2\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$\text{Fo (sometimes } \tau) = \frac{\alpha t}{L^2}$	$\frac{\text{Physical time}}{\text{Thermal diffusion time}}$
Froude number	$\text{Fr} = \frac{V}{\sqrt{gL}} \left(\text{sometimes } \frac{V^2}{gL}\right)$	$\frac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$\text{Gr} = \frac{g\beta\vert\Delta\vert TL^3\rho^2}{\mu^2}$	$\frac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$\text{Ja} = \frac{c_p(T - T_{\text{sat}})}{h_{fg}}$	$\frac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$\text{Kn} = \frac{\lambda}{L}$	$\frac{\text{Mean free path length}}{\text{Characteristic length}}$



Lewis number	$\mathrm{Le} = \dfrac{k}{\rho c_p D_{AB}} = \dfrac{\alpha}{D_{AB}}$	$\dfrac{\text{Thermal diffusion}}{\text{Species diffusion}}$
Lift coefficient	$C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$	$\dfrac{\text{Lift force}}{\text{Dynamic force}}$
Mach number	$\mathrm{Ma}\ (\text{sometimes } M) = \dfrac{V}{c}$	$\dfrac{\text{Flow speed}}{\text{Speed of sound}}$
Nusselt number	$\mathrm{Nu} = \dfrac{Lh}{k}$	$\dfrac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$
Peclet number	$\mathrm{Pe} = \dfrac{\rho L V c_p}{k} = \dfrac{LV}{\alpha}$	$\dfrac{\text{Bulk heat transfer}}{\text{Conduction heat transfer}}$
Power number	$N_P = \dfrac{\dot{W}}{\rho D^5 \omega^3}$	$\dfrac{\text{Power}}{\text{Rotational inertia}}$
Prandtl number	$\mathrm{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\mu c_p}{k}$	$\dfrac{\text{Viscous diffusion}}{\text{Thermal diffusion}}$
Pressure coefficient	$C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\dfrac{\text{Static pressure difference}}{\text{Dynamic pressure}}$
Rayleigh number	$\mathrm{Ra} = \dfrac{g\beta \lvert \Delta T \rvert L^3 \rho^2 c_p}{k\mu}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$



Reynolds number	$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$	$\frac{\text{Inertial force}}{\text{Viscous force}}$
Richardson number	$\mathrm{Ri} = \frac{L^5 g \Delta\rho}{\rho \dot{V}^2}$	$\frac{\text{Buoyancy force}}{\text{Inertial force}}$
Schmidt number	$\mathrm{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$	$\frac{\text{Viscous diffusion}}{\text{Species diffusion}}$
Sherwood number	$\mathrm{Sh} = \frac{V L}{D_{AB}}$	$\frac{\text{Overall mass diffusion}}{\text{Species diffusion}}$
Specific heat ratio	k (sometimes γ) $= \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$
Stanton number	$\mathrm{St} = \frac{h}{\rho c_p V}$	$\frac{\text{Heat transfer}}{\text{Thermal capacity}}$
Stokes number	Stk (sometimes St) $= \frac{\rho_p D_p^2 V}{18 \mu L}$	$\frac{\text{Particle relaxation time}}{\text{Characteristic flow time}}$
Strouhal number	St (sometimes S or Sr) $= \frac{f L}{V}$	$\frac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Weber number	$\mathrm{We} = \frac{\rho V^2 L}{}$	Inertial force